Solutions to JEE Main - 3 | JEE - 2024

PHYSICS

SECTION-1

1.(B) $t=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\times 490}{9.8}}=10s \quad \therefore \quad R=150\times 10=1500\,\text{m}$

2.(C) When accelerating upwards,

$$N-mg=ma \Rightarrow N=mg+ma$$

When accelerating downwards,

$$mg-N'=ma \Rightarrow N'=mg-ma$$

$\therefore \frac{N'}{N}=\frac{g-a}{g+a}=\frac{10-2}{10+2}; \qquad N'=\frac{2}{3}N=80$

3.(A) $\bar{a}_{rel}=g-g=0$

$\bar{v}_{rel}=\text{constant}$

$\therefore$ Path of projectile as seen by another projectile is a straight line.

4.(B) Acceleration of a point at the tip of the blade

= centripetal acceleration $=\omega^2 R=(2\pi f)^2 R$

$$=\left(2\times\frac{22}{7}\times\frac{1200}{60}\right)^2\times\frac{30}{100}=4740\text{ m/sec}^2$$

5.(B) The tension in the string between P and Q accelerates double the mass as compared to that between Q and R. Hence, tension between P and Q = 2 × tension between Q and R.

6.(A) Limiting value of static frictional force on $B=\mu_s m_B g$

$f_L=0.5\times 7\times 10=35N \qquad \Rightarrow a_{\max}$ of $A=1\text{ m/s}^2$

$\Rightarrow$ Blocks move togetehr if $0<F<42N$

As $F=100N$

and the two bodies will move in the same direction, (i.e., of applied force) but with different accelerations. Here, force of friction $\mu_k m_B g$ will oppose the motion of B which will cause the motion of A. So, the equation of motion of block B will be,

$$F-\mu_k m_B g=m_B a_B \qquad \dots \text{(i)}$$

i.e., $a_B=\frac{(F-\mu_K m_B g)}{m_B}=\frac{(100-0.4\times 7\times 10)}{7} \qquad =\frac{100-28}{7}=\frac{72}{7}=10.3\text{ m s}^{-2}$

While for body A, $f_K=m_A a_A$,

i.e., $a_A=\frac{\mu_K m_B g}{m_A}=\frac{0.4\times 7\times 10}{35}=0.8\text{ m s}^{-2}$

7.(D) $4mg-mg=ma$

$\Rightarrow$ $a=3g$

8.(D) The particle is actually projected with velocity u at an any angle 60° from the horizontal and it hits the wall when it is moving horizontally.

Hence $t = \frac{u\sin\theta}{g} = \frac{u\sqrt{3}}{2g}$

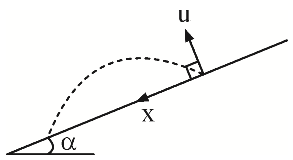
9.(A) Suppose u be the velocity of projection. The greatest range on level ground is given by:

$$R = \frac{u^2}{g} = 6000 \text{ m}$$

Maximum range up the inclined plane (say R_1) is given by:

$$R_1 = \frac{u^2}{g(1+\sin\beta)} = \frac{6000}{1+\sin 30°} = 4000 \text{ m.}$$

10.(C) $0 = ut - \frac{1}{2} g\cos\alpha t^2$



$t = \frac{2u}{g\cos\alpha}$

$R = \frac{1}{2} g\sin\alpha t^2 = \frac{1}{2} g\sin\alpha \times \frac{4u^2}{g^2\cos^2\alpha} = \frac{2u^2}{g} \times \frac{\tan\alpha}{\cos\alpha}$

11.(D) Let, $u_x = 3$ m/s, $a_x = 0$; $u_y = 0$, $a_y = 1$ m/sec^2 and $t = 4$sec

If v_x and v_y be the velocities after 4 sec respectively, then

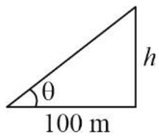
$v_x = u_x + a_x t = 3 \text{ m s}^{-1}$ and $v_y = u_y + a_y t = 0 + 1\times 4 = 4 \text{ m s}^{-1}$

$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = 5$ m/s

Angle made by the resultant velocity w.r.t. direction of initial velocity, i.e., x-axis, is

$$\beta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{4}{3}\right).$$

12.(B) $R = \frac{u^2\sin 2\theta}{g} \Rightarrow 100 = \frac{(500)^2\sin 2\theta}{g}$



$\sin 2\theta = \frac{1000}{500\times 500} = \frac{1}{250} \quad \Rightarrow \quad$ (θ is a small angle)

$\tan\theta = \frac{h}{100} \Rightarrow h = 100\tan\theta = \frac{100}{500} = 0.2m, \; = 20cm$

13.(D) Given that at any instant t

$x = kt$ and $y = kt - k\alpha t^2$

If u be the initial velocity of projectile and α be the angle of projection, then

$x = (u\cos\alpha)t$ and $y = (u\sin\alpha)t - \frac{1}{2}gt^2$

Comparing above equations, with the given equation, we get;

$u\sin\alpha = k$ and $g = 2k\alpha$

If t_m be time taken to reach maximum height, then at max. height, $v_y = 0$

i.e., $v_y = \frac{dy}{dt} = k - 2k\alpha t_m = 0 \quad \therefore \quad t_m = \frac{1}{2\alpha}$

Hence, time of flight, $T = 2t_m = 2\left(\frac{1}{2\alpha}\right) = \frac{1}{\alpha}$

Now, maximum height attained, $H = \frac{(u\sin\alpha)^2}{2g} = \frac{k^2}{4k\alpha} = \frac{k}{4\alpha}$

14.(D) Weight of one-third chain = friction on two third chain

$\frac{mg}{3} = \frac{\mu 2mg}{3} \Rightarrow \mu = \frac{1}{2}$

15.(C) Time taken by the cart to cover 80 m,

$$t = \frac{s}{v} = \frac{80}{30} = \frac{8}{3}\text{sec}$$

We have, $T_f = \frac{2u}{g} = \frac{8}{3} \Rightarrow u = \frac{40}{3}$ m/s

16.(C) **(A)** $a = \frac{2mg - mg}{3m} = \frac{g}{3}$ **(B)** $a = \frac{2mg - mg}{m} = g$

(C) $a = \frac{2mg}{m} = 2g$ **(D)** $a = \frac{2mg}{3m} = \frac{2g}{3}$

17.(A) $\tan\theta = \frac{u\sin\theta}{u\cos\theta} = \frac{2}{1}$

The desired equation is, $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

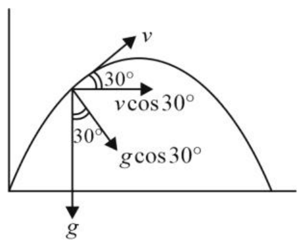
$$= x \times 2 - \frac{10x^2}{2(\sqrt{2^2+1^2})^2\left(\frac{1}{\sqrt{5}}\right)^2} \quad \text{or} \quad y = 2x - 5x^2.$$

18.(C) Let v be the velocity of particle when it makes 30° with horizontal. Then,

$$v\cos 30° = u\cos 60°$$

or $v = \frac{u\cos 60°}{\cos 30°}$

$= \frac{(20)(1/2)}{(\sqrt{3}/2)} = \frac{20}{\sqrt{3}}$ m/s



Now, $g\cos 30° = \frac{v^2}{R}$

or $R = \frac{v^2}{g\cos 30°} = \frac{(20/\sqrt{3})^2}{10(\sqrt{3}/2)} = 15.4\,\text{m}$

19.(B) When the horizontal range is maximum the maximum height attained is $R/4$. Hence, co-ordinates of the point = (200, 100)

20.(B) As $\tan 37° > \mu$, so block slides down with acceleration

$$a = g\sin\theta - \mu g\cos\theta$$

$$= 10\left(\frac{3}{5}\right) - (0.5)10\left(\frac{4}{5}\right)$$

$$= 6 - 4 = 2 \text{ m/s}^2$$

$\sin 37° = \frac{6}{\ell} \quad \Rightarrow \quad \ell = 10\text{m}$

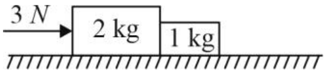
$\therefore \quad v^2 = u^2 + 2as$

$= 0 + 2(2)(10) \ = 40$

$\therefore \quad v = 2\sqrt{10}$ m/s

SECTION – 2

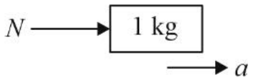
1.(1)



Common acceleration of the system, $a = \frac{3}{2+1} = 1\ \text{ms}^{-2}$

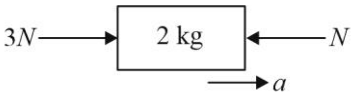
Let N be the force of contact between the two blocks.

The free body diagram of 1 kg block is as shown in the figure:



$N = 1 \times a = 1 \times 1 = 1N$

The free body diagram of 2 kg block is as shown in figure:



$3 - N = 2a$

$N = 3 - 2a = 3 - 2 \times (1) = 1N$

2.(27) Here mass of the ball $m = 10g = 0.01$ kg

Let v be the velocity of the ball.

$\therefore$ Change in momentum $= mv - (-mv) = 2mv$

$F_{avg} = \frac{\Delta \bar{p}}{\Delta t} \Rightarrow \Delta \bar{p} = F_{avg} \times t = 5.4 \times 0.1 \quad = 0.54$

$v = \frac{0.54}{2m} = \frac{0.54}{0.02} = 27$

3.(2) Given: $m_1 = 1$ kg, $m_2 = 6$ kg and $m_3 = 3$ kg

If a is the acceleration of the system

$m_1 a = T_1 - m_1 g$

$m_2 a = T_2 - T_1$

$m_3 a = m_3 g = T_2$

Adding, $a(m_1 + m_2 + m_3) = (m_3 - m_1)g$

$\therefore \quad a = \frac{(m_3 - m_1)g}{(m_1 + m_2 + m_3)} = \frac{(3-1) \times 10}{1+6+3} = 2\ \text{m s}^{-2}.$

4.(15) Speed in horizontal direction remains constant during whole journey because there is no acceleration in this direction.

So, $u_h = 5\ \text{m s}^{-1}$

In vertical direction:

Hence, the speed with which he touches the cliff B is:

$$v = \sqrt{v_h^2 + v_V^2} = \sqrt{25+200} = \sqrt{225} = 15 \text{ ms}^{-1}$$

5.(24) Given that; $\theta = 2t^3 + 0.5$

$\therefore \quad \dfrac{d\theta}{dt} = 6t^2 = 6\times(2)^2 = 24$ rad/sec

CHEMISTRY

SECTION-1

1.(A) According to modern periodic table, the physical and chemical properties of the elements are periodic function of their atomic numbers and after every 2 – 8 – 8 – 18 – 18 – 32 the similar properties elements repeats.

2.(C) Atomic number = 120

IUPAC name = Unbinilium

Symbol = Ubn

Alkaline earth metal (s-Block elements)

Electronic configuration $\Rightarrow {}_{118}[\text{Og}]8\text{s}^2$

3.(B)

List-I		**List-II**	
(P)	Same value of electron gain enthalpies	**(II)**	Ar and Kr
(Q)	Same values of electronegativity on Pauling scale	**(I)**	N and Cl (EN = 3)
		(IV)	K and Rb (EN = 0.8)
(R)	Same nature of oxides (amphoteric)	**(III)**	Al and As
(S)	Same chemical reactivity	**(IV)**	K and Rb

4.(A) For alkali metals, as we move down the group, the size of atom increases, easy ionisation of metals. Thus down the group, reactivity of metal increases. For halogens, as we move down the group, the size of atom increases, low electron affinity. Thus down the group reactivity of non-metals decreases.

5.(C)

(P)	Order of I.E.	**(I)**	Be > B
		(III)	N > P
		(IV)	I < Xe
(Q)	Order of electron gain enthalpy	**(II)**	O < S
(R)	Order of valence	**(II)**	O < S
		(IV)	I < Xe
(S)	Order of ability to form $p\pi - p\pi$ multiple bonds to itself	**(III)**	N > P

6.(B)

(P)	p-Block elements	**(II)**	Six columns
(Q)	d-Block elements	**(III)**	Group 3 to 12
(R)	s-Block elements	**(I)**	Two columns
(S)	f-Block elements	**(IV)**	Two rows of elements at the bottom of the periodic table, having group number 3

7.(C) Oxidising property increases in a period and decreases in the group.

$\Rightarrow$ It depends on atomic size, electronegativity and hydration energy

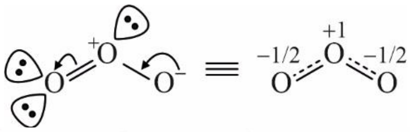
$\Rightarrow$ Oxidising property order $\Rightarrow$ F > O > Cl > N

8.(A) Lithium unlike other alkali metals form compounds with pronounced covalent character, the other member of this group forms predominant ionic compounds.
Li and Mg have similar properties, which is referred as diagonal relationship in the periodic properties.

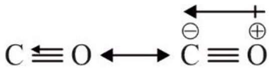
9.(C) In general, in a group on moving down a group, the size increases, thus I.E decreases.
But $IE_1 \Rightarrow$ Al < Ga, because of poor shielding of 3d in Ga.

10.(C) Mendelevium $= {}_{101}Md$

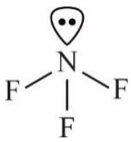
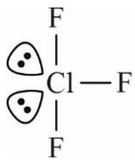
* IUPAC name = Unnilunium, Symbol = Unu
* It is an actinoid element
* It is a radioactive element
* It is discovered by Glenn T. Seaborg

11.(D) $O_3 \Rightarrow$ 

12.(D)  ; $110 < BL < 121$

13.(B) Carbon monoxide 
Oxygen is more electronegative than carbon meaning electron density is going to be pulled in the direction of oxygen, away from carbon. But as contribution of co-ordinated π-bond is more compared to electronegativity factor in the overall polarity of the molecule. So, direction of dipole moment in CO is from 'O' to 'C'.

14.(D)

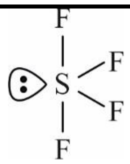
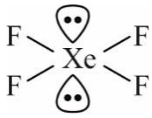
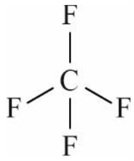
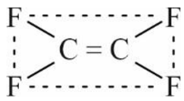
	NF_3	ClF_3
		
Hybridization =	sp^3	sp^3d
Shape =	Trigonal pyramidal	See-saw
% p =	75%	60%
Type of bond =	sp^3-p	sp^3d-p

ClF_3 has two types of bond length.
Valency of N and Cl is '3'.

15.(B) $^{-}O-Cl=O \qquad {}^{-}O-\underset{\underset{O}{\|}}{Cl}=O \qquad {}^{-}O-\overset{\overset{O}{\|}}{\underset{\underset{O}{\|}}{Cl}}=O$

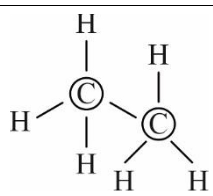
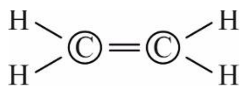
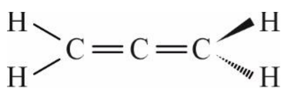
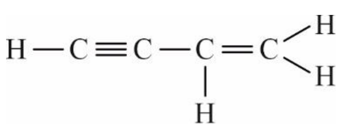
16.(B) 'pi' bond has nodal plane.

17.(B)

(P)	SF_4		**(II)**	See-saw
(Q)	XeF_4		**(IV)**	Square planar
(R)	CF_4		**(III)**	Tetrahedral
(S)	C_2F_4		**(I)**	Rectangular

18.(A) Resonance hybrid is hypothetical structure, average of two or more canonical structure. And being more stable, energy of resonance hybrid is least in compare to canonical structures.

19.(C)

CH_3-CH_3		sp^3 hybrid carbon Each 'C' atom is tetrahedral Non-planar structure
$CH_2=CH_2$		sp^2 hybrid carbon Each 'C' atom is trigonal planar HCH bond angle = 120° Planar
$CH_2=C=CH_2$		Non-planar HCH bond angle = 120° CCC bond angle = 180° sp hybridised 'C' is most electronegative
$HC\equiv C-CH=CH_2$		Planar sp hybridised 'C' is most EN CCC bond angle = 180° HCH bond angle = 120°

20.(D) Hybrid orbitals are mono-centric.

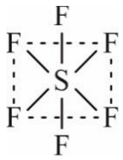
SECTION – 2

1.(9) $Cl_xO_y \Rightarrow Cl_2O_7;\ x + y = 2 + 7 = 9$

2.(8) Order of negative electron gain enthalpy

16^{th} group	–	S > Se > Te > Po > O
17^{th} group	–	Cl > F > Br > I > At
And	–	O < At

- 'O' atom has least negative value of electron gain enthalpy
- Number of proton of 'O' atom = 8

3.(12) $SF_6 \Rightarrow$ 

Number of 90° F – S – F bond angles = 12

4.(9) SF_4, ClF_3, XeF_2, XeF_4, SO_2, XeO_3, SO_2Cl_2, I_3^- & IF_3^{2-} have expanded octet.

5.(4)

KNO_3	K^+	$^-O-N=O$, $N \downarrow O$
$NaBF_4$	Na^+	$F-\underset{\mid}{\overset{\mid}{B}}\leftarrow F^-$ (with F above and below B)
KI_3	K^+	$I-I\leftarrow I^-$
NH_4Cl	Cl^-	$H-\underset{\mid}{\overset{\mid}{N}}\rightarrow H^+$ (with H above and below N)

MATHEMATICS

SECTION-1

1.(A) Let $S = \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + ...$

$\Rightarrow \quad 3S = \frac{3}{1.4} + \frac{3}{4.7} + \frac{3}{7.10} + ...$

$\Rightarrow \quad 3S = \frac{4-1}{1.4} + \frac{7-4}{4.7} + \frac{10-7}{7.10} + ...$

$\Rightarrow \quad 3S = \left(\frac{1}{1} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{10}\right) + ...$

$\Rightarrow \quad 3S = \frac{1}{1} \Rightarrow S = \frac{1}{3}$

2.(A) $\sum_{r=1}^{120} \frac{1}{\sqrt{r+1} + \sqrt{r}} = \sum_{r=1}^{120} (\sqrt{r+1} - \sqrt{r}) = (\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3}) + ... + (\sqrt{121} - \sqrt{120})$

$= \sqrt{121} - \sqrt{1} = 10$

3.(C) $t_n = n(n)! = (n+1)! - n! \Rightarrow \sum_{n=1}^{20} t_n = 21! - 1$

4.(C) Here $x^4 > 0, \frac{9}{x^4} > 0$

$\therefore$ Using AM – GM inequality, we get,

$$\frac{x^4 + \frac{9}{x^4}}{2} \geq \sqrt{x^4 \times \frac{9}{x^4}}$$

$\Rightarrow \quad x^4 + \frac{9}{x^4} \geq 6$

5.(C) $\frac{\sin 2x + \sin 3x + \sin 4x}{\cos 2x + \cos 3x + \cos 4x} = \frac{(\sin 2x + \sin 4x) + \sin 3x}{(\cos 2x + \cos 4x) + \cos 3x}$

$$= \frac{2\sin 3x \cos(-x) + \sin 3x}{2\cos 3x \cos(-x) + \cos 3x}$$

$$= \frac{\sin 3x[2\cos x + 1]}{\cos 3x[2\cos x + 1]} \qquad [\because \cos(-\theta) = \cos\theta]$$

$$= \tan 3x$$

6.(C) Given $\frac{x}{\sin\theta} = \frac{y}{\sin\left(\theta + \frac{2\pi}{3}\right)} = \frac{z}{\sin\left(\theta - \frac{2\pi}{3}\right)} = \lambda$ (say)

$\Rightarrow \quad x + y + z = \lambda\left\{\sin\theta + \sin\left(\theta + \frac{2\pi}{3}\right) + \sin\left(\theta - \frac{2\pi}{3}\right)\right\} = \lambda\left\{\sin\theta + 2\sin\theta\cos\frac{2\pi}{3}\right\} = 0$

7.(A) $\frac{1-abc}{a}, \frac{1-abc}{b}, \frac{1-abc}{c}$ are in A.P. $\quad \Rightarrow \quad \frac{1}{a} - bc, \frac{1}{b} - ca, \frac{1}{c} - ab \rightarrow$ A.P.

8.(C) $t_n = S_n - S_{n-1} = \frac{n(n+1)(n+2)}{6} - \frac{(n-1)n(n+1)}{6} = \frac{n(n+1)}{2}$

9.(A) Let A & D be first term and common difference respectively of corresponding AP

Then $\quad A + (m-1)D = \frac{1}{n} \qquad \ldots$(i)

$A + (n-1)D = \frac{1}{m} \qquad \ldots$(ii)

$\Rightarrow \quad D = \frac{1}{mn}, A = \frac{1}{mn}$

$\therefore \quad T_{mn} = \frac{1}{mn} + (mn-1)\frac{1}{mn} = 1$

10.(C) $\frac{a}{1-r} = 7 \qquad \ldots(1)$

$\frac{ar}{1-r^2} = 3 \qquad \ldots(2)$

solving $r = 3/4$ and $a = 7/4$

$\Rightarrow \quad a + r = 5/2$

11.(A) Let the G.P. be a, ar, ar^2 and terms of A.P. are $A+d, A+4d, A+8d$

then $\dfrac{ar^2-ar}{ar-a}=\dfrac{(A+8d)-(A+4d)}{(A+4d)-(A+d)}=\dfrac{4}{3}$

$\Rightarrow \quad r=\dfrac{4}{3}$

12.(B) $\sin x=\pi$ not possible

$\cos x=\dfrac{-2}{3}\rightarrow$ two solution in $[0, 2\pi]$

13.(C) $\dfrac{1}{a_1},\dfrac{1}{a_2},\dfrac{1}{a_3},......$ in A.P. $\quad\Rightarrow\quad \dfrac{1}{a_{10}}=\dfrac{1}{a_1}+9d \Rightarrow \left(\dfrac{1}{15}-\dfrac{1}{5}\right)=9d \Rightarrow \dfrac{-2}{15\times 9}=d$

So, $a_n<0 \quad\Rightarrow\quad \dfrac{1}{a_n}<0 \quad\Rightarrow\quad \dfrac{1}{a_1}+(n-1)d<0$

$\Rightarrow \quad \dfrac{1}{5}+(n-1)\left(\dfrac{-2}{15\times 9}\right)<0\Rightarrow 27-2(n-1)<0\Rightarrow 2(n-1)>27\Rightarrow n>14.5$

14.(C) $\cos 20^\circ\cos 40^\circ\cos 60^\circ\cos 80^\circ$

$=\cos 60^\circ[\cos 20^\circ\cdot\cos 40^\circ\cdot\cos 80^\circ]$

$=\left(\dfrac{1}{2}\right)\left[\dfrac{1}{4}\cos(3\times 20^\circ)\right]=\dfrac{1}{16}$

15.(B) Now, $S_\infty=1+\dfrac{4}{3}+\dfrac{7}{3^2}+........\infty;$

$\dfrac{1}{3}S_\infty=\dfrac{1}{3}+\dfrac{4}{3^2}+........\infty;$

$\Rightarrow \quad \dfrac{2}{3}S_\infty=1+\dfrac{3}{3}+\dfrac{3}{3^2}+........\infty;$

$\Rightarrow \quad \dfrac{2}{3}S_\infty=1+\dfrac{1}{1-1/3}\Rightarrow S_\infty=\dfrac{3}{2}\left(1+\dfrac{3}{2}\right)=\dfrac{15}{4}$

16.(A) Given, $\dfrac{\sin x-\sin 7x}{\cos 7x-\cos x}=\tan 6x$

$\Rightarrow \quad \dfrac{-2\cos 4x\sin 3x}{-2\sin 4x\sin 3x}=\tan 6x$

$\Rightarrow \quad \cot 4x=\tan 6x$

$\Rightarrow \quad \tan\left(\dfrac{\pi}{2}-4x\right)=\tan 6x$

$\Rightarrow \quad 10x=\dfrac{\pi}{2}\Rightarrow x=\dfrac{\pi}{20}$

17.(A) As, $\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\alpha-\sin^2\beta$

$=\cos^2\alpha-(1-\cos^2\beta)$

$=\cos^2\alpha+\cos^2\beta-1=s-1$

18.(C) $\dfrac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)}=\dfrac{7}{5}\Rightarrow\dfrac{\sin(\alpha+\beta)\cos(\alpha-\beta)}{\sin(\alpha-\beta)\cos(\alpha+\beta)}=\dfrac{7}{5}$

Using componendo and dividendo, we get

$\dfrac{\sin 2\alpha}{\sin 2\beta}=\dfrac{12}{2}=6$

19.(B) Let 1^{st} term be 'a' and common ratio be 'r'

$$\frac{a(1-r^{101})}{1-r}=125$$

$$\sum_{i=1}^{101}\frac{1}{a_i}=\left(\frac{1}{a_1}+\frac{1}{a_2}+........+\frac{1}{a_{101}}\right)$$

$$=\frac{1}{a}\left(\frac{1-r^{101}}{1-r}\right)\cdot\frac{1}{r^{100}}=\frac{125}{(ar^{50})^2}=\frac{125}{625}=\frac{1}{5}$$

20.(A) Let $b=ar, c=ar^2$

$$x=\frac{a(1+r)}{2}$$

$$y=\frac{ar(1+r)}{2}$$

$$\left(\frac{a}{x}+\frac{c}{y}\right)=\left\{\frac{a\times 2}{a(1+r)}+\frac{ar^2\times 2}{ar(1+r)}\right\}=\left\{\frac{2}{1+r}+\frac{2r}{1+r}\right\}=2$$

SECTION – 2

1.(96) $T_r=\dfrac{\left(\dfrac{r(r+1)}{2}\right)^2}{r^2}=\dfrac{r^2+1+2r}{4}\Rightarrow S_9=\dfrac{1}{4}\left[\dfrac{9\times 10\times 19}{6}+9+9(10)\right]=96$

2.(4) H.M. $\dfrac{2\alpha\beta}{\alpha+\beta}=\dfrac{2(\text{product})}{\text{sum}}=\dfrac{2(8+2\sqrt{5})}{(4+\sqrt{5})}=4$

3.(14) First common term = 23

Common difference $=7\times 4=28$

Last term $\leq 407 \quad \Rightarrow \quad 23+(n-1)\times 28\leq 407 \quad \Rightarrow \quad 28(n-1)\leq 384$

$\Rightarrow n\leq 13.71+1 \quad \Rightarrow \quad n\leq 14.71 \quad \Rightarrow \quad n=14$

4.(1) $\sin^6 x+\cos^6 x=(\sin^2 x)^3+(\cos^2 x)^3=(\sin^2 x+\cos^2 x)^3-3\sin^2 x\cos^2 x(\sin^2 x+\cos^2 x)$

$$=1-3\sin^2 x\cos^2 x=1-\frac{3}{4}(\sin 2x)^2\Rightarrow(\sin^6 x+\cos^6 x)_{\max}=1$$

5.(4) $S=\dfrac{7}{2^2\,5^2}+\dfrac{13}{5^2\,8^2}+\dfrac{9}{8^2\,11^2}+......$

$$3S=\frac{21}{2^2\,5^2}+\frac{39}{5^2\,8^2}+\frac{57}{8^2\,11^2}+......$$

$$3S=\sum_{r=1}^{10}\frac{(3r+2)^2-(3r-1)^2}{(3r-1)^2(3r+2)^2}$$

$$3S=\sum_{r=1}^{10}\left(\frac{1}{(3r-1)^2}-\frac{1}{(3r+2)^2}\right)$$

$$3S=\frac{1}{2^2}-\frac{1}{2^{10}}$$

$$3S=\frac{2^8-1}{2^{10}},\ S=\frac{85}{1024}=\frac{m}{n}$$

Hence, $(n-12m)=4$